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Analogy in Volta's Exact Natural Philosophy

Thomas Edison, who once symbolized invention as Einstein did science, was a master in the deployment of analogy. He made his first successes in telegraphy by considering parallels to a system of pumps, pipes and water wheels, which he understood better than electrical components. The electric telegraph then became the starting line from which, by analogy, he raced to the dictaphone, phonograph, and motion pictures, among many other things. Without “a logical mind that sees analogies”, he warned, no one should set up as an inventor.¹ The opinion, if not the dictum, was not new. As Aristotle wrote of the skills needed by a poet, “The greatest thing by far is to be a master of metaphor ... since a good metaphor implies an intuitive perception of the similarity in dissimilars”.² Edison recognized the implied parallels between creativity in writing and in invention. “Shakespeare! ... He would have been an inventor, a wonderful inventor, if he had turned his mind to it ... His originality in the way of expressing things has never been approached”.³

Historians tend to lose sight of the functioning of analogy in early modern science. That is because the dissimilars invoked in analogy then often crossed what are now disciplinary lines. If the historical actor did not make his analogy explicit, it is easily missed by historians who identify themselves, and divide up their fields, anachronistically. This division may be impossible to erase. Historians trained in the sciences cut up fields as their textbooks do. The operation is sanctified and reinforced by their prime research tool, the bibliographies issued annually by the U.S. History of Science Society in its journal *Isis*, which classify the literature under anachronistic rubrics like “sixteenth century: biological sciences”. Most seasoned historians know that this label is nonsense: neither the concept nor the word “biology” existed in the 17th century, nor did people then privilege a division of

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¹ T.P. HUGHES, “Designing, Developing, and Reforming Systems”, *Daedalus*, 127:4 (1998), 215-32, on p. 216; P. ISRAEL, *Edison: A Life of Invention*, (New York, 1998), p. 68 (quote), pp. 143-4, 251, 292-4.

² ARISTOTLE, *De poetica*, chap. 22 (1459a5-8).

³ EDISON, in ISRAEL, cit. 1, p. 29.

history by century.⁴ The myths of *Isis* nonetheless continue to define our sub-disciplines and constrain our thought.

An example of the resulting damage is the neglect of the interlocked set of analogies employed by Lavoisier, Volta, and Galvani during the 1770s and 1780s to advance their several lines of research. The analogies, which ran through phlogiston to animal electricity, covered fields that now come under the divided attention of historians of chemistry, physics, meteorology and physiology. Considering their work as a whole, however, we can see important, even essential, links among the new chemistry, the artificial electricity of the laboratory and lecture hall, the natural electricity of the atmosphere and animal bodies, the discoveries of Galvani, and the invention of the pile.⁵ The natural knowledge of the late 18th century was more of a piece than our histories suggest.

The similarity detected among dissimilars in science and in poetry is usually qualitative and pictorial. Another sort of analogizing characterized quantitative analysis. Here quantities representing disparate physical ideas may be treated in parallel mathematically. Thus Volta's analyses of the electrification of a condenser, the evaporation of liquids, and the working of the pile, which have the same mathematical structure, imply, and may well have been stimulated by, analogies among charge, quantity of vapor and current, on the one hand, and tension, temperature and emf, on the other. No doubt similar parallels will be perceived elsewhere in his work by historians who do not set inappropriate disciplinary divisions within it.

A further discussion of the damage done by anachronistic divisions of the subject matter of early-modern science makes up the first part of what follows. Then comes an account of Volta's broad qualitative analogizing and an analysis of his quantitative parallels. It will appear that Volta reasoned in the style, if not in the form, of differential calculus, which lent itself to the transcription of one of his favorite techniques, both in experiment and in exposition. This technique, the repetition of small steps, may be appreciated in the manipulations of the electrophorus and the condensatore and depreciated in his prolix style, which abounds in phrases like "a disk 4. 6. 8. or more inches in diameter", "a charge of 10. 20. 50. 100. degrees", "an increase of one degree, then of 2. 3. 4. etc."⁶ True to the parallel with which we began, Volta's genius expressed itself similarly in the laboratory and the study; he wrote in the same incremental way in which he reasoned and experimented.

⁴ "Biology" was coined around 1800; century reckoning developed during the 17th century and did not become standard in historical writing until the 18th. A. PICHOT, *Histoire de la notion de vie*, (Paris, 1993), p. 58; and J. BURKHARDT, *Die Entstehung der modernen Jahrhundertrechnung: Ursprung und Ausbildung einer historiographischen Technik von Flacius bis Ranke*, (Gippingen, 1971), pp. 43, 49, 62-3, 79, respectively.

⁵ J.L. HEILBRON, *Electricity in the 17th and 18th Centuries: A Study of Early Modern Physics* (Berkeley, 1979), reprinted, with a new preface, (New York, 1999), preface.

⁶ *VO*, I, p. 543; II, p. 146; III, pp. 235-6, 239, 243, 247.

1. A Self-Inflicted Injury

The disciplinary blindfolds that inhibit recognition of the play of analogy have several baleful consequences for the historiography of early-modern science, besides neglect of analogy. For example, the question, recently actively debated in the historical literature, whether Lavoisier was a chemist or physicist or both, is an artifact of false categories.⁷ To his contemporaries, Lavoisier was a *physicien*, which should be translated “natural philosopher”, not “physicist”. The incorrect translation gave rise to the incorrect associations that fed the debate over Lavoisier’s disciplinary identification. The English word “physicist” did not exist in the 18th century, nor did its modern connotations. It was coined around 1840, about the same time as “scientist”, to express a new concept for which the world lacked words. (*Scienziato* and *scientifique* used as nouns are also 19th-century coinages). As labels for new roles, the new names met resistance from people who persisted in seeing themselves as natural philosophers – as non-technical, broadly knowledgeable investigators of nature, not as money-seeking, narrowly based professionals.⁸ Lavoisier was an experimental natural philosopher with a strong interest in the combinations of matter.

The pseudo-problem of Lavoisier’s disciplinary identity has given life to the pseudo-question, whether he effected a revolution in or into chemistry. If he were a chemist, the argument runs, the revolution would be internal; if a physicist, external. The question misses the reality that both natural philosophy and chemistry (understood as the study of the combinations of matter) were revolutionized around 1770 by the discovery of the different gas types. Their study called on apparatus traditional in the chemical arts (like glassware) and on instruments used in experimental natural philosophy (like electrical machines); and also on brand new items, like the pneumatic trough and the gazometer.⁹ Pneumatics, the study of gases, did not belong exclusively or entirely to any discipline with a modern name; it figured in textbooks of chemistry and, at least as prominently, in texts on natural philosophy. Indeed, the discovery of the gas types appears as the most important advance since Galileo in Johann Carl Fischer’s *Geschichte der Physik*, published in

⁷ A. DONOVAN, “Lavoisier and the Origins of Modern Chemistry”, *Osiris*, 4 (1988), pp. 214-31, on pp. 215, 221-2, 226-8; C.E. PERRIN, “Research Traditions, Lavoisier, and the Chemical Revolution”, *Osiris*, 4 (1988), pp. 53-81, on pp. 62-3, 79-80; and “Chemistry as Peer of Physics: A Response to Donovan and Melhado on Lavoisier”, *Isis*, 81 (1990), pp. 259-70, on pp. 262, 265-7; E.M. MELHADO, “Metzger, Kuhn, and Eighteenth-Century Disciplinary History”, in G. FREUDENTHAL, ed., *Etudes sur Hélène Metzger*, (Paris, 1989), pp. 11-34, on p. 116.

⁸ S. ROSS, “Scientist: The Story of a Word”, *Annals of Science*, 18 (1962), pp. 65-85.

⁹ M. TRUCHOT, “Les instruments de Lavoisier”, *Annales de chimie*, 18 (1879), pp. 289-319; J. PARASCANDOLA and A. IHDE, “History of the Pneumatic Trough”, *Isis*, 60 (1969), pp. 351-60; T.H. LEVERE, “Lavoisier: Language, Instruments, and the Chemical Revolution”, in T.H. LEVERE and W.B. SHEA, eds., *Nature, Experiment and the Sciences*, (Dordrecht, 1990), pp. 207-23, on pp. 218-9.

eight volumes between 1801 and 1808; and an authoritative modern historian takes the determination of the weight and properties of the air as the true origin of experimental physics.¹⁰ A further indication of the very considerable overlap between “physics” and “chemistry” around 1800 is that both the *Annales de chimie*, founded in 1789, and the *Annalen der Physik*, founded in 1790, enlarged their titles if not their scope to include “physique” and “Chemie” in 1816 and in 1819, respectively.

Lavoisier’s case is particularly interesting because he has become the exclusive property of historians of chemistry who guard their turf by fixing the research agenda and applying an inappropriately narrow definition of “physics”. The tactic works because historians of physics are happy to leave a subject now one of the most scholastic in the historiography of science to those who want it. Most of the few efforts that have been made to integrate Lavoisier’s scientific persona have been made by historians of chemistry, whose work is seldom cited outside their sub-discipline.¹¹ Lavoisier’s case is by no means unique. His contemporaries whose work overlapped his, like Henry Cavendish, J.C. Wilcke, and Volta, also have had their personalities split by modern historians. Here an important exception must be made for intellectual biographies that review an entire *oeuvre*.¹² As the organization of this volume shows, however, most knowledgeable students of Volta divide him up according to their sub-disciplinary affiliations.

An unusually influential misuse of the labels of disciplinary history is the distinction, now classic, made by T.S. Kuhn between what he called mathematical and experimental traditions, or classical and Baconian sciences. The first achieved coherence and steady progress in antiquity: Kuhn had in mind geometry, astronomy, geometrical optics, and mechanics (the statics of Archimedes). These are the sciences that underwent revolution during the 17th century. The other set of sciences, represented by electricity, magnetism, and physical optics gained data during the 17th and 18th centuries but little in the way of progressive theory. They were revolutionized and mathematicized around 1800 by practitioners of the maturer classical sciences. Thus classical physics was born.¹³ Kuhn’s suggestive dichotomy

¹⁰ M. GLIOZZI, “Le origini della fisica sperimentale: La determinazione del peso specifico dell’aria”, *Periodico di matematiche*, 11 (1931), pp. 1-10, on p. 1.

¹¹ H. GUERLAC, “Chemistry as a Branch of Physics: Laplace’s Collaboration with Lavoisier”, *Historical Studies in the Physical Sciences*, 7 (1976), pp. 183-276; F. ABBRI, *Le terre, l’acqua, le arie: La rivoluzione chimica del settecento*, (Bologna, 1984); cf. LEVERE, cit. 9, pp. 210, 218-9; A. LUNDGREN, “The Changing Role of Numbers in Eighteenth-Century Chemistry”, in T. FRÄNGSMYR et al., eds., *The Quantifying Spirit of the Eighteenth Century*, (Berkeley, 1990), pp. 245-66, on pp. 245-6, 257-63.

¹² E.g., C.W. OSEEN, *Johan Carl Wilcke Experimental-Fysiker*, (Uppsala, 1939); G. POLVANI, *Alessandro Volta*, (Pisa, 1942); C. JUNGNIKEL and R. MCCORMACH, *Cavendish*, (Philadelphia, 1996).

¹³ T.S. KUHN, *The Essential Tension: Selected Studies in Scientific Tradition and Change*, (Chicago, 1977), pp. 31-6, 41-2, 61; J.L. HEILBRON, “A Mathematicians’ Mutiny, with Morals”, in P. HORWICH, ed., *World Changes: Thomas Kuhn and the Nature of Science*, (Cambridge, 1993),

omits the complication that, like most things, the make-up of mathematical sciences changed radically between Antiquity and the Enlightenment. By the end of the 18th century, mathematics included calendrics, cartography, fortification, metrology, navigation and surveying, as well as astronomy, mechanics, geometrical optics and mathematics in the modern sense. Its practitioners had little interest in trying to quantify the subject matter of natural philosophy.

Quantitative physics arose in a rearrangement and transformation of what the *Royal Society* of London called “natural knowledge”. We should not think of a conquest of physics by mathematicians already in possession of its classical parts but of an entirely different process: the simultaneous seeping into many areas of natural knowledge of the *esprit géométrique* supposed to characterize the later Enlightenment.¹⁴ As we read in the preface that the Marquis de Condorcet and Sylvestre-François Lacroix added to their edition of Leonhard Euler’s *Lettres à une princesse d’Allemagne*, published in 1787, “mathematics is, and ought to be, part of any good education”.¹⁵

Contemporaries recognized the interpenetration of the physical sciences and their rapprochement with mathematics. Here is what they said: “chemistry has been brought much closer than hitherto to physics” (Lavoisier, 1799); “[chemistry] has become the inescapable aid and companion of physics” (J.A. Pictet, 1785); “the true *physicien* is one who speaks the language of chemistry” (R.J. Haüy, 1806); “[Haüy’s subject] is fisica chimica” (M. Landriani, 1781); “the objects of applied mathematics ... belong in themselves to physics” (J.S.T. Gehler, 1795); “applied mathematics is made up really of individual parts of physics” (G.W. Lichtenberg, 1794); “the elementary notions of mathematics [are] absolutely essential to anyone who wants to learn physics” (M.J. Brisson, 1800); “[physics consists of] the choicest parts [of applied mathematics plus] the most necessary topics of chemistry” (J.C. Fischer, 1800).¹⁶ In short, as a thesis defended at the University of Pavia in 1790 put it, “*mathematica studia ad universam physicarum rerum scientiam sunt penitus necessaria*”.¹⁷

Again, one must beware of investing the old words with the meanings they now carry. The expanding, quantitative physics of 1800 differed from the physics of 1900 and, *a fortiori*, from today’s, not only in scope but also in objective. Very little of the older physics, however quantitative it might have been, allowed the successful prediction or retrodiction of quantitative experimental results from mathematically

pp. 107-12.

¹⁴ FRÄNGSMYR et AL., cit. 11, provide many examples.

¹⁵ L. EULER, *Lettres à une princesse d’Allemagne*, 3 vols., M.J.A.N. CARITAT, Marquis de CONDORCET and S.F. LACROIX, eds., (Paris, 1787-9), I, p. vii.

¹⁶ References in J.L. HEILBRON, *Weighing Imponderables and Other Quantitative Science around 1800 (Historical Studies in the Physical Sciences*, 24:1 (1993), suppl.), pp. 26-7, 30-1; to which add M.J. BRISSON, *Dictionnaire de physique*, 2nd ed., 6 vols., (Paris, 1800), I, pp. vii.

¹⁷ “*Assertiones tres ex physica publice propugnatae a Sebastiano Scaramuzza ...*”, cited in S. SERENA, *S. Gregorio Barbarigo e la vita spirituale e culturale nel suo seminario di Padova*, 2 vols., (Padova, 1963), I, pp. 74-5.

expressed principles. Around 1800, people perfected their measuring instruments to quantify, control and inventory, without being able often to link their quantities via equations derived from general principles. Indeed, much of their work was a search for concepts that they could quantify usefully and relate phenomenologically. To avoid misleading associations, their occupation might be termed “exact natural philosophy” rather than “quantitative physics”.

The search for quantitative concepts was peculiarly challenging around 1800 because the quantities sought typically were occult, that is, not immediately given to the senses. Whereas in the exemplar of mathematical science (astronomy) a basic element of the theory and the prime object of measurement (the angular separation of celestial bodies) are one and the same, the quantities of chief interest to physics or natural philosophy, like fire, heat, light, and electricity, had no immediately given measures. Part of Volta’s genius was his ability to find and refine measurable correlates or proxies for theoretical constructs like electric charge and tension.¹⁸

What set loose the quantifying spirit and transformed the body of natural knowledge? The answer lies in the requirements of the nation states and their military, in the growth of commerce and manufactures, in the compulsion to control rapidly multiplying information about the world, and in the influence of instrumentalist philosophies. Some of these forces were at work in the Austrian government of Lombardy’s patronage of the University of Pavia. Volta’s brand of exact natural philosophy with its cornucopia of new instruments was the sort of science enlightened governments liked to support.

2. The Role of Analogy

Volta’s Exact Natural Philosophy

Volta stood with Cavendish, A.C. Coulomb, J.A. Deluc, and H.B. de Saussure in the van of the exact measurers of his time. They understood that the improvement of natural philosophy required many hands, which, however, would work at cross purposes unless they possessed instruments that were not only exact but also robust and intercomparable. All of them spent much time and trouble calibrating instruments and developing protocols for their use. The importance of their endeavor was widely recognized. For example, in 1776 the *Royal Society* of London set up a committee to determine why its thermometers never agreed. Cavendish was its chairman and Deluc one of its members. The committee traced the trouble to differences in the placement of the bulb and in the ambient pressure when marking the boiling point of water, and they designed an elaborate procedure to achieve standardization. During the 1770s and 1780s, Deluc and Saussure took equal or greater pains perfecting hygrometers and barometers. The fat books in which they

¹⁸ Cf. SOCIETÀ IDROELETTRICA COMACINA, “L’opera del Volta nello sviluppo dell’industria elettrica”, in COMO-MUNICIPIO, ed., *Como ad Alessandro Volta nel II centenario della nascita*, (Como, 1945), pp. 217-32, on p. 220.

described their corrections and wrote down their formulas were not for the lackadaisical, qualitative, laissez-faire virtuosi of yesteryear but for the modern exact natural philosopher. Volta rated them masterpieces.¹⁹

Volta qualifies for membership in this band of exact natural philosophers for his eudiometer, which made the determination of the respirability of the air reliable; his straw electrometer, which registered equal quantities of electricity by equal divisions on the scale; and the condensatore and condensing microscope, which magnified small charges to the point of detection by the electrometer. All these instruments indicate another important characteristic of Volta's practice: the incarnation of his (and others') discoveries in usable instruments. Thus his eudiometer improved greatly on its predecessors by employing hydrogen as the working gas, simplifying the mechanism for introducing the test gas, and detonating the mixture electrically. His condensatore and condensing electroscope descend from his electrophorus, itself a useful instrument incorporating the insights he and others had derived from the dissectible condenser.²⁰

The eudiometer and the condensatore also incorporate Volta's technique of repetitive manipulations. His first crude eudiometric measurement began with eight volumes of common air and one of inflammable air (hydrogen). He sparked the mixture, added a second, third and fourth volume of hydrogen, sparking each time successfully apart from the last mixture, which he could not explode. His perfected technique began with a non-explosive mixture of hydrogen with air or oxygen. He then added small quantities of the one or the other, dollop by dollop, until the spark flashed. That did not end the investigation for Volta. He continued, measure by measure, until he had made a mixture that would not explode. With four parts of hydrogen, for example, he could detonate as little as two and a quarter parts of air, and also, 3, 4, 5 ... and more, as many as 54 parts; and once he mentioned a series of experiments that ended in unexplosive mixtures after trials with 13, 14, 20, and even 100 parts of air.²¹

The condensatore was an electrophorus run backwards. Volta attached a source of weak electricity, like an atmospheric probe in serene weather, to the shield of an insulated, unelectrified electrophorus. Because of the high capacity and low tension of the instrument when assembled, the shield acquired most of the probe's charge. Volta would then touch the insulated base of the electrophorus, giving it a small charge; raise the shield; break the connection with the probe; discharge the shield; reassemble the apparatus; and repeat the steps, until the charge on the base plate became large enough to register on an electroscope.²²

¹⁹ HEILBRON, cit. 5, p. 76; POLVANI, cit. 12, pp. 201, 208.

²⁰ HEILBRON, *ibid.*, pp. 416-9, 457, 493; POLVANI, *ibid.*, *passim*; W.A. OSMAN, *Annals of Science*, 14 (1958), pp. 215-42, on pp. 221-31.

²¹ *VO*, VI, pp. 182-3, 189-90, 298 f; OSMAN, *ibid.*, pp. 221, 225-31, 235-8. Volta first fired gas mixtures electrically in December 1776 (*VO*, VI, pp. 49-50).

²² A. VOLTA, *Philosophical Transactions*, 72:1 (1782), pp. vii-xxxiii, esp. §§ 3, 9, 33-4, 49 (*VO*, III, pp. 272-6, 286-7, 294); *VO*, I, pp. 420-4, 435-7 (1796-7).

Volta's notion that the exact natural philosopher had a duty to incorporate discoveries in reliable useful instruments may be seen in the investigations that produced the battery. He had elaborated his definitive answer to Galvani by 1796. It persuaded many among the handful of people then knowledgeable about galvanism. Why did he feel compelled to go further? In a note in a recent issue of *Isis*, this question is raised and answered in the unfortunate manner of our time. It is because Volta wished to multiply virtual witnesses, to generate a fact, and to claim jurisdiction over the phenomena.²³ In plain English, according to this interpretation Volta invented the pile to appeal directly to the senses of people unable to reproduce his subtle experiments on contact electricity. No doubt Volta would have liked to convince everyone who noticed his ideas. But that was a secondary matter. He needed the pile primarily to realize his program of reducing his discoveries to art, of completing and demonstrating his mastery of a new effect by materializing it in a reliable and useful instrument.

Volta developed the quantifiable concepts materialized in his instruments and measurements by developing the analogies between the imponderable, springy fluids supposed to be the causes of the phenomena of electricity, magnetism, heat, light and combustion. The discovery of the gas types, brought to center stage in the theater of natural philosophy by Joseph Priestley's revelations in the early 1770s, strengthened the system of imponderables. The set of gases seemed to reproduce, in a more material way, the set of weightless fluids, to have similar elasticity and to interact in the same manner. For example, the phlogistic explanation of the combination of hydrogen and oxygen into water, favored by Priestley, Cavendish and Volta, was perfectly homologous with Franklin's explanation of the discharge of the Leyden jar.

A favorite subject of measurement for Volta, Deluc and Saussure was the interaction of the fluids, especially heat, with the gases. The evaporation of water presented a particularly delicate problem in both natural philosophy and exact description. What was the mechanism of evaporation? How did new heat divide its effect between the production of more vapor and the dilation of vapor already present? Volta lacked the principles for deciding the mechanism and the division. That did not stop him from quantifying the problem. He proceeded by analogy to a case he knew how to treat exactly: the charging in slow motion of the plates of a condenser. Here the increments of electrical fluid and tension, on the one hand, corresponded to increases in heat matter and temperature, on the other.

When he came to try to incorporate his understanding of galvanism in a usable instrument, Volta turned eventually to repeating many small disks of dissimilar metals. Again he lacked the principles to analyze the pile and again he turned to analogy. The outcome – though here the documents do not permit certainty – was similar to the analysis that J.B. Biot, a competent mathematician and *physicien* of

²³ J. MERTENS, "Shocks and Sparks: The Voltaic Pile as a Demonstration Device", *Isis*, 89 (1998), pp. 300-11, on pp. 307, 311.

Laplace's school, gave after Volta had demonstrated his battery in Paris. Let us look more closely at these cases.

Conjugate Conductors

Volta's manuscripts abound with sheets similar to that transcribed on figure 1, which portrays two identical facing insulated conductors.²⁴ Volta played with social metaphors to describe the situation. Apart, each plate was a recluse (*solitario*); together they formed a union (*connubio*) or association (*consorzio*) or, the more intimate relation he settled on, a marriage (*conduttori conjugati*).²⁵ To meter this intimacy, Volta attached a thread to each plate as indicated in the figure. The greater the height to which the thread rises, the greater the charge on its conductor. The purpose of the arrangement is to charge a condenser (that is, the conjugated conductors) by adding electrical fluid to the plates one after the other, drop by drop. Volta performed the summation with experimental apparatus and with a pencil. His manuscripts suggest that he enjoyed multiplication (figure 2).

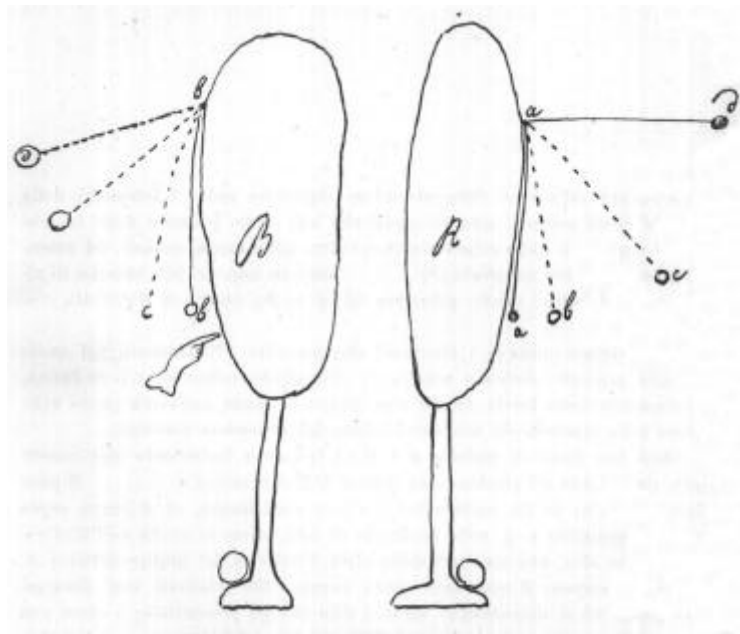


Figure 1 Conjugate conductors. *VO*, III, p. 166.

²⁴ *VO*, III, pp. 248-58 (1782); *Cart. Volt.*, I.13.

²⁵ *Ibid.*, pp. 233, 234 (1778-80).

100. efendo ¹⁰⁰ Del punto A al punto (1) ¹⁰⁰
 2. it. ¹⁰⁰ Acquisita alla subscrizione ¹⁰⁰

99-1	72490	74715	80078	100
99-2	652404	642435	726746	801
99-3	652404	642435	726746	801
99-4	71711	43968	80232	801
99-5	65030	605712	721436	80209
99-6	65030	605712	721436	80209
99-7	73228	4930	80209	80209
99-8	650032	71424	80209	80209
99-9	650032	71424	80209	80209
99-10	72490	76508	80209	80209
99-11	645030	704094	80434	80434
99-12	645030	704094	80434	80434
99-13	71053	4478	80434	80434
99-14	630471	40002	80434	80434
99-15	630471	40002	80434	80434
99-16	630471	40002	80434	80434
99-17	630471	40002	80434	80434
99-18	630471	40002	80434	80434
99-19	630471	40002	80434	80434
99-20	630471	40002	80434	80434
99-21	630471	40002	80434	80434
99-22	630471	40002	80434	80434
99-23	630471	40002	80434	80434
99-24	630471	40002	80434	80434
99-25	630471	40002	80434	80434
99-26	630471	40002	80434	80434
99-27	630471	40002	80434	80434
99-28	630471	40002	80434	80434
99-29	630471	40002	80434	80434
99-30	630471	40002	80434	80434
99-31	630471	40002	80434	80434
99-32	630471	40002	80434	80434
99-33	630471	40002	80434	80434
99-34	630471	40002	80434	80434
99-35	630471	40002	80434	80434
99-36	630471	40002	80434	80434
99-37	630471	40002	80434	80434
99-38	630471	40002	80434	80434
99-39	630471	40002	80434	80434
99-40	630471	40002	80434	80434
99-41	630471	40002	80434	80434
99-42	630471	40002	80434	80434
99-43	630471	40002	80434	80434
99-44	630471	40002	80434	80434
99-45	630471	40002	80434	80434
99-46	630471	40002	80434	80434

Figure 2 The arithmetic of charging by droplets. VO, III, facing p. 256.

The game goes like this. Touch a large Leyden jar to plate A , raising its potential to that of the jar (unity, say) and giving it a charge a . This charge raises the tension of plate B to some amount $x < 1$ that is larger the smaller the separation of B from A . Volta emphasized that the separation must be large enough that no electrical fluid passes between the plates, and small enough that B feels A 's presence. Volta observed that the pressure of an enclosed gas can be increased either by augmenting its amount or by heating it. The electrical fluid in B behaves under A 's influence "just as the air in a receiver, maintained at its natural density, would increase in expansive force when heated". There is this difference, however: B 's increased tension amounts to a virtual charge, to an *elettricità potenziale*.²⁶ He activated this potential by touching (earthing) B . It acquired a negative charge $-ax$, which is just enough, following the rule $Q = C T$, to drop the tension in B to zero. (The charge a on A creates a potential $+x$ at B ; a charge $-ax$ on B creates a tension $-x$ there). Now, just as the charge a on A produced a tension x in B , the charge $-ax$ produces a tension $-x^2$ in A ; touching the bottle to A causes a flow of charge ax^2 to bring the net tension of A back to unity.

The charge $a(1 + x^2)$ on A excites a tension in B . How much? Reasoning in his habitual incremental style, Volta supposed that the equilibrium in the previous step held, that the charge a on A maintained at tension 1 still equilibrated the charge $-ax$ on B maintained at tension 0. Hence only the increment of charge ax^3 acted to produce the new tension in B ; assuming the same decrement x , this new tension would be x^3 . Touching B , Volta replaced this tension by an increment of charge $-ax^3$, which, acting on A , diminished its tension by the increment $-x^4$. Another kiss from the bottle brought A 's tension to 1 and its charge to $a(1 + x^2 + x^4)$; owing to the preceding equilibrium, only the last increment in this series caused a new tension, x^5 in B .²⁷ The series ends as follows:

$$Q_A = a + ax + ax^2 + ax^4 + \dots = a/(1 - x^2) = 5.26a \text{ or } 10.25a$$

$$Q_B = -ax - ax^3 - ax^5 - \dots = -ax/(1 - x^2) = -4.74a \text{ or } -9.74a$$

for $x = 0.9$ or $x = 0.95$, respectively.

This neat account, in which the instrumental manipulations exactly parallel the algebra, not only displays the operation of the rule $Q = C T$ in slow motion and the strong effect of the separation of the plates on the charge stored, but also shows the limit of validity of Franklin's teaching that a Leyden jar contains no more electrical fluid when charged than when not.²⁸

²⁶ Ibid., pp. 240 (quote), 244 (texts of 1778-80).

²⁷ Ibid., pp. 248-9 (text of c. 1782).

²⁸ B. FRANKLIN, *Experiments and Observations on Electricity*, I.B. COHEN, ed., (Cambridge, Mass., 1941), pp. 180, 237-8; F.U.T. AEPINUS, *Tentamen theoriae electricitatis et magnetismi*, (St. Petersburg, 1759), a book Volta valued highly (*VO*, III, p. 326), had emphasized (on pp. 82-7) that the charges on the jar could not cancel exactly.

Volta confirmed his theory of incremental charging by measurement of the angular displacement of the threads attached to the conjugate conductors separated by various distances. Naturally he wanted to know how x behaved as he gradually moved the plates apart. Let d be their separation. Volta arranged his results in tables like figure 3, in which the first column gives the tension in A (T_A), the second d , the third his experimental determination of $T_B = x T_A$, and the fourth the calculated value of T_B . The calculation appears to have been made using the formula $x = 12/(12 + d)$. Volta did not exhibit it or its justification. In not attempting to derive this phenomenological formula from principles, Volta practiced the exact physics of his time. That does not mean that he accorded it no wider significance. Doubtless it strengthened his conviction that Coulomb's "law" could not be universally true.²⁹

Dilatation and Evaporation

Between 1791 and 1795, Volta made extensive measurements on the expansion of air and water vapor by heat. This work covered a key area in physical science, the interaction of the imponderables with the gases. The critical point was evaporation. Did it occur, as Deluc and Lavoisier insisted, via a true combination of caloric and water, or in another way? Volta disliked the combination theory apparently because he could not believe that matter could destroy the elasticity of heat as it could that of fixed air. "I don't resort to depriving such an element of its innate and essential calorific action". Instead he accounted for latent heat as a consequence of change of heat capacity on change of state. Perhaps his reluctance to allow the destruction of heat through binding derived from a parallel to electricity, which, though it could be held in place by ordinary matter, did not combine with it to produce a new substance.³⁰ In any case, Lavoisier's demonstration of the composition of water in 1782 changed Volta's mind. He then allowed that water vapor was a true gas, a combination of water with heat and/or phlogiston. These views date from 1784, from Volta's anonymous contributions to the Italian edition of Macquer's *Dictionnaire de chimie*.³¹

Volta may have been drawn into measurement of the dilation of air and vapor by the discordant numbers in circulation in the late 1780s. The authoritative and careful Deluc had obtained for ϵ , the percentage increase in the volume of air at constant pressure per degree Réaumur, 1/215; but a dozen other values, from Priestley's 1/85

²⁹ HEILBRON, cit. 5, pp. 461-2, 475 (*VO*, V, pp. 78-9, 81-3); P. HEERING, "The Replication of the Torsion Balance Experiment: The Inverse Square Law and its Refutation by Early 19th-Century German Physicists", in CH. BLONDEL and M. DÖRRIES, eds., *Restaging Coulomb: Usages, controverses et répliques autour de la balance de torsion*, (Firenze, 1994), pp. 47-66, on pp. 57-8, 63-4; and L. FREGONESE, "Two Different Scientific Programs: Volta's Electrology and Coulomb's Electrostatics", *ibid.*, pp. 85-98, on pp. 85-90.

³⁰ Cf. F. SEBASTIANI, "La memoria voltiana intorno al calore", *Physis*, 23 (1981), pp. 89-113, on pp. 90, 96-7.

³¹ *VO*, VII, pp. 19-20, 87-8, 101; POLVANI, cit. 12, pp. 189, 215-17.

A elettrizzato attua				B		
a distanza				sper. ^{ze}	secondo	
di linee					il calcolo	
gr. 13	—	1	—	gr. 11 $\frac{1}{2}$	—	12
14	—	2	—	12	—	12
15	—	3	—	12	—	12
16	—	4	—	12	—	12
15	—	6	—	10	—	10
15	—	8	—	9	—	9
16 $\frac{1}{2}$	—	10	—	9	—	9
16	—	12	—	8	—	8
14	—	16	—	6	—	6
15	—	18	—	6	—	6
18	—	24	—	6	—	6
18	—	24	—	6 $\frac{1}{4}$	—	6
15	—	18	—	6	—	6
14	—	16	—	6	—	6
16	—	12	—	8	—	8
16 $\frac{1}{2}$	—	10	—	9	—	9
15	—	8	—	9	—	9
15	—	6	—	10 $\frac{1}{4}$	—	10
16	—	4	—	12	—	12
15	—	3	—	12 $\frac{1}{2}$	—	12
14	—	2	—	12 $\frac{1}{2}$	—	12
13	—	1	—	12 $\frac{1}{2}$	—	12

Figure 3 Decrease of induction with distance. *VO*, III, p. 257.

to Saussure's 1/235, competed with Deluc's, a situation that Volta, with his concern for reproducibility and interest in eudiometry, must have thought disgraceful.³² His first experiments on air date from 1791. He enclosed a volume of air in a bulb mounted on a glass stem containing oil or water or mercury, and submerged the whole, stem side open and down, in a water bath. A small mercury thermometer dipped in the bath to record its temperature as Volta heated it from the freezing to the boiling point of water. He observed, in his habitual incremental manner, "not omitting ... to note the volume of air not only for every 10 or 20 degrees, but for every degree of heat, or, at least, every other, both when the heat in the bath was increasing and also – with more patience – when the heat decreased, in its subsequent spontaneous cooling".³³ The uncommonness of this compulsive measuring may be inferred from the gross interpolations and extrapolations made by Gay-Lussac and Dalton, respectively, from their few measurements to obtain their values of ϵ .³⁴ Volta's result, $\epsilon = 1/216$, confirmed Deluc's.

Volta succeeded in obtaining a reproducible value of ϵ by drying his air thoroughly. This necessity surprised him greatly. The slightest contamination with water ruined the measurements, since some of the liquid existed as vapor whatever the temperature. "Water therefore abundantly transforms into elastic vapor at a heat several degrees below boiling, at the ordinary pressure of the air; which perhaps no one would have believed". His explanation of the surprising result employed two of his favorite ideas. One is continuity. The prevailing notion that water suddenly converted from a liquid to a vapor at the boiling point made no sense to him. "One [that is, Volta] was astonished, and should have been, at the abrupt passage from the simple dilation of water to its expansion as an air; but now we know that here too is a progression and a simple law of continuity for the simple formation of this vapor". A second idea Volta invoked was a mechanism of contact and affinity: air helps to transform warmed water into vapor. The text from which these quotations come dates from 1791.³⁵ The further development of Volta's ideas about evaporation coincided with the investigations that produced his theory of the electrification of metals by contact.

Volta's ongoing study of the dilation of vapors employed the same sort of summation as the one he had used in analyzing the conjugate conductors. In a manuscript dating from around 1795, Volta separated the dilation into two parts, the production of new vapor and the expansion of it and the vapor already present.³⁶ (The former he identified as a "purely physical effect", the latter as one "appropriately chemical", arising from the solution of water in caloric; a usage that

³² POLVANI, *ibid.*, p. 201; HEILBRON, *cit.* 16, pp. 90-4; OSMAN, *cit.* 20, p. 240 (*VO*, VI, p. 298).

³³ *VO*, VII, p. 370; POLVANI, *cit.* 12, p. 204.

³⁴ POLVANI, *ibid.*, pp. 210, 230.

³⁵ *VO*, VII, pp. 326-7 (quotes); T.S. FELDMAN, *The History of Meteorology, 1750-1800: A Study in the Quantification of Experimental Physics*, PhD thesis, (University of California, Berkeley, 1983), pp. 101-11.

³⁶ *VO*, VII, pp. 423-5, 457-60.

shows sufficiently the danger of applying modern disciplinary labels to 18th century science). Figure 4 is a representative data sheet; when reduced to symbols, it shows strong similarities to the electrical case. Volta began with b units of water vapor at 0° R and supposed that a additional units evaporated in raising the temperature by 2° R. The total vapor became $Q(2) = a + b$; and the pressure, $(a + b)(1 + 2\varepsilon)$, assuming, as he did, the important rule that all elastic fluids have, more or less, the same coefficient of expansion at constant volume.

What happens when the temperature goes up to 4° ? Here is the typically Voltaic answer: the amount of vapor present, $a + b$, is augmented by some proportion, $x > 1$, of the increment a only. So:

$$Q(4^\circ) = a + b + ax; \quad Q(6^\circ) = a + b + ax + ax^2; \quad \text{etc.}$$

The corresponding pressures are:

$$Q(4^\circ) (217/213); \quad Q(6^\circ) (219/213); \quad \text{etc.}$$

Volta's numbers, $\alpha = 2$, $\beta = 0.4545$ (later amended to 0.44), $x = 1.108$, $\varepsilon = 1/213$, agreed well with his experiments. The construction of the series cannot be justified by modern theory and the experimental results, recorded for every other degree, differ by as much as 25 percent from modern ones.³⁷ The interest of the matter for present purposes, however, is not Volta's accuracy, but his method. In his thinking as well as in his mathematics, heating the tube by a temperature ΔT develops a quantity of new vapor, just as touching either of the conjugate conductors annuls a tension ΔT and develops a quantity of new charge. Volta used a geometric progression to represent the increments of vapor with stepwise increase in temperature just as he had represented changes of charge with a stepwise creation and destruction of tension. In both cases the progression is on the increment, not on the total, and the current value of the variable is a sum over the increments. Like the electrical case, the pneumatic provided endless opportunities for arithmetic (figure 5).

The Pile

In 1796, in his first letter to Gren, Volta inventoried the various arrangements for exciting a frog or an electroscope (figure 6). The strongest effect came from two dissimilar metals (represented by capital letters) in contact with a moist conductor (represented by a lower-case letter), as in diagram number 1. Volta attributed the effect to an electromotive force that caused one of the pair of metals to give or release, and the other to receive or take, a small quantity of electrical fluid. (These terms derive from Franklin's theory of positive and negative electricity).³⁸ In the strongest case, silver was the donor and zinc the acceptor. The same give and take

³⁷ POLVANI, cit. 12, pp. 224-8.

³⁸ FRANKLIN, cit. 28, pp. 180-3.

Term. Reaum.	Quantità materiale di vapore		Aumento di elasticità		Pressione risultante
Gradi 10	4,8215	×	223	=	5,046
	0,7598		213		
12	5,5813	×	225	=	5,896
	0,8419		213		
14	6,4232	×	227	=	6,845
	0,9328		213		
16	7,3560	×	229	=	7,908
	1,0335		213		
18	8,3895	×	231	=	9,098
	1,1451		213		
20	9,5346	×	233	=	10,477
	1,2688		213		
22	10,8034	×	235	=	11,916
	1,4058		213		
24	12,2092	×	237	=	13,59
	1,5576		213		
26	13,7668				
	1,7258				
28	15,4926				
	1,9124				
30	17,4050	×	243	=	19,865
	2,1188		213		
32	19,5238				
	2,3476				
34	21,8714				
	2,6011				
36	24,4725	×	249	=	28,33
	2,8821		213		
38	27,3546				
	3,1933				
40	30,5479	×	253	=	36,285
	3,5382		213		
42	34,0861				
	3,9201				
44	38,0062				
	4,3437				
46	42,3499	×	259	=	51,48
	4,8128		213		

Figure 4 Increase of vapor pressure with temperature. *VO*, VII, p. 459.

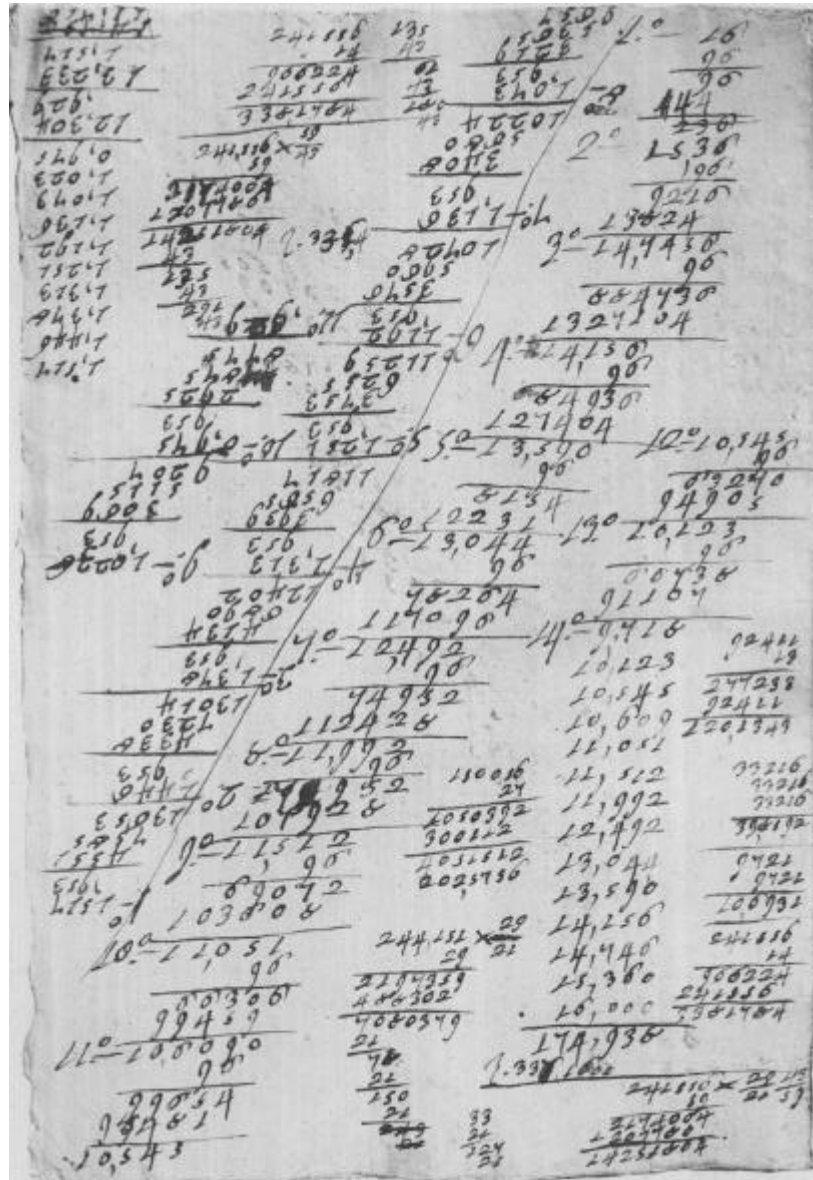


Figure 5 Calculation of the decrease in vapor pressure, assumed to be geometric, for every degree from 80° R to 0° R. The pressure at 80° is taken as 16, the decrement per degree as 0.96. *VO*, VII, after p. 464.

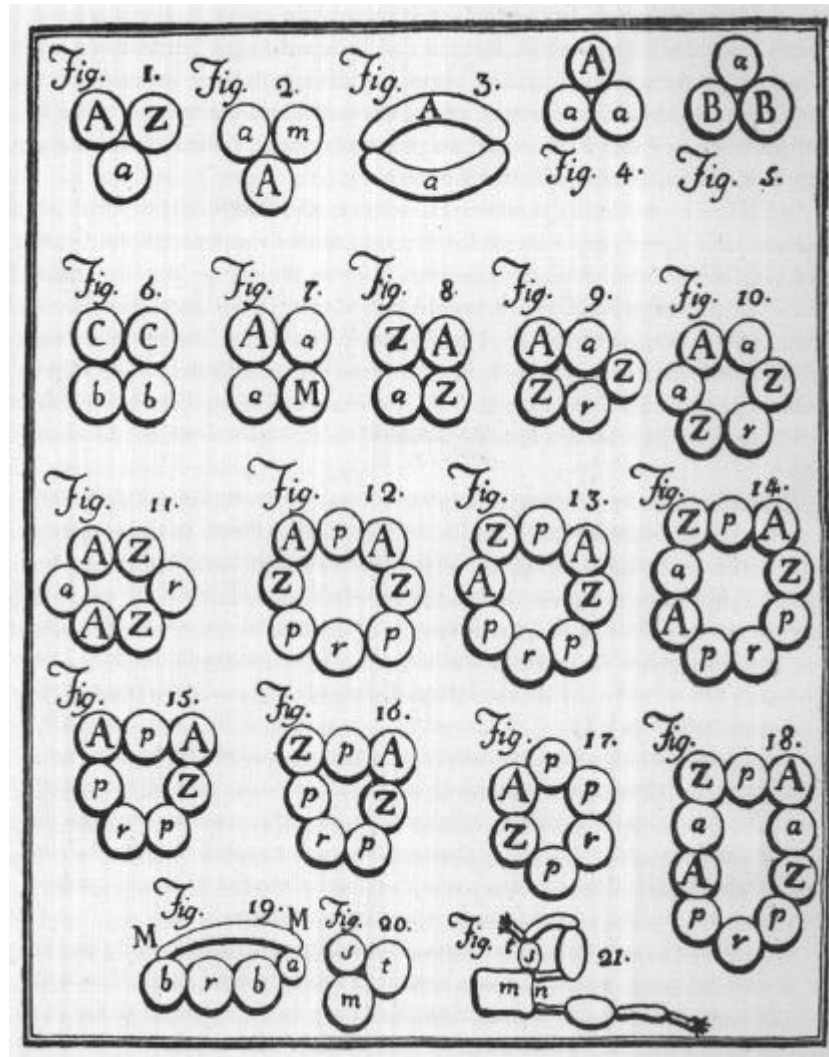


Figure 6 Arrangements of conductors of the first and second kind. *VO*, I, p. 398.

occurred at the boundaries between metallic and moist conductors, at much lower intensities; here the metal, or conductor of the first kind, always gave and the moist or second-class conductor always received.³⁹

By April 1798, when he wrote to Aldini, Volta had worked out the direction of the displacement of the electrical fluid in contacts between various sorts of conductors. Volta says that he succeeded in enhancing the effects beyond his expectations by increasing the size of the metal plates and moistened cards to a diameter of two to three inches.⁴⁰ Some time thereafter he began a search for another way to amplify the metallo-electric effect and to convert it into a practical instrument for the “perpetual” production of electricity. What we know of Volta's methods in other cases allows a guess at how he advanced from the letter to Aldini to the letter to Banks, from triumphing over Galvani to announcing the battery.

Among the arrangements Volta had drawn for Gren in 1796 was a horizontal pile with two elements (number 13 on figure 6). We can infer Volta's understanding of its operation from his letter to Aldini of 1798. In the first couple, *A* gives *Z* a quantity of electric fluid *Q*, and *Z* gives *p* a much smaller amount *q*; a net (*Q* – *q*) is shifted across *p*, suffering some loss perhaps owing to the poor conductivity of the moist conductor (figure 3). Let the factor of diminution be *x*; then the net effect of the triplet *A*, *Z*, *p* is to push charge *x* (*Q* – *q*) into the second silver couple *A'*, *Z'*. (*A'* gives something to *p*; *q'* represents the net charge displaced to the right through the moist conductor taking into account its contact emf with *Z* and *A'*).

Meanwhile *A'* had given *Z'* the usual dose *Q*, and *Z'* had given *p'* its ration *q*; so that, if there were a third couple, it would pass on the quantity:

$$(Q - q')x + (Q - q')x^2$$

from the left and donate (*Q* – *q'*) to the right.

The series would continue:

$$Q + (Q - q')x + (Q - q')x^2 + \dots = (Q - q')/(1 - x) + q'.$$

The algebraic form is close to the analysis of the conjugate plates and so is the physical idea – the evolution of new charge at each step to be added to the charge present. Just as the ultimate sources of the plates' charges are an inexhaustible Leyden jar and the earth, so the pile electrifies as described only when grounded. (An insulated pile cannot have a net charge). The same physical idea – the stepwise production of a new quantity of active material from an inexhaustible source to be added to the material already in play – is precisely that of the analysis of evaporation and dilation.⁴¹ The concept of a practically inexhaustible source played a large part

³⁹ *VO*, I, pp. 395-406, 419-20, 544-7 (giving and receiving), 548-51 (second-class conductors). The rule is not invariable; A. WÜLLNER, *Lehrbuch der Elementarphysik*, 4th ed., 4 vols., (Leipzig, 1882-6), IV, pp. 462-3.

⁴⁰ VOLTA, *VO*, I, pp. 540, 542-4 (area).

⁴¹ Cf. *VO*, I, pp. 419-20: “Questa forza o tendenza produce, se il cerchio è altronde compito per mezzo di conduttori umidi, una corrente, un giro continuo di esso fluido, che va ... dall'argento

in Volta's thinking and inventing. It occurs perhaps first in the electrophorus, figures in fictional Leyden jars and recurs in the pile.

The choice of the repeat elements A , Z , p was suggested, if not dictated, by the principle of Volta's explanation of galvanic experiments. His belief that by repeating them in an appropriate pattern he could integrate their power was strengthened by his knowledge that even a fish could do it. As he mentioned several times in his letter to Banks, the form of the pile was exactly that of the electrical organ of the torpedo. In fact, the new electromotor contained the working ghosts of two different animals. Galvani's frogs, metamorphosed into the soaked cardboard bits, provided a conducting path between the metal pairs that made possible the stepwise increase in the pile's potential, and the electric fish, virtually present in the stacking, epitomized in its incremental repetition of electrical elements precisely Volta's preferred method of experiment and analysis (figure 7).

It is likely, as Giuliano Pancaldi has argued, that J.W. Nicholson's design, published in 1799, of an artificial torpedo, made of a series of tiny condensers connected in parallel, provided an important impulse toward the development of the pile. Two aspects of Nicholson's design would have helped to convince Volta that the electrical fish offered the model to be imitated in amplifying the emf of a single metal pair. For one, Nicholson placed the source of the fish's electricity in its columnar musculature, not in its brain, where Volta (as well as Galvani) then located it. Secondly, Nicholson gave a calculation, based upon his measurement of the electricity developed in the splitting of thin flakes of mica, to show that an effect comparable to the torpedo's shock could be obtained from a large stack of very thin plates electrified in the manner of mica. That was the sort of argument that impressed Volta. Proceeding then with "his brain in [his] fingertips", Volta doodled and tinkered his way to the battery (figure 7).⁴²

When Volta demonstrated his pile and crown of cups in Paris in 1801, the *Institut* set up a committee consisting of its best calculators and experimental philosophers to examine the phenomena and find their principles. Biot wrote their report, which the formidable committee, from Laplace and Coulomb down, signed. Biot transcribed Volta's insights into algebra and treated the insulated as well as the grounded case. In one respect, however, Biot's elegant rephrasing was less exact than Volta's semi-quantitative discussion, for he vacillated among "*état*", "*quantité du fluide*", and "*tension*" as the item augmented by the action of the pile, and he omitted internal resistance and the action at the contact between the metals and the

allo stagno, e da questo per la via del conduttore o conduttori umidi ritorna all'argento per ripassare nello stagno, ec.: se il circolo non è compiuto, se i metalli trovansi isolati, un'accumulazione di detto fluido elettrico nello stagno a spese dell'argento ... elettricità picciola è vero ... ma che pure sono giunto finalmente a rendere, più che non avrei sperato, sensibile ...".

⁴² G. PANCALDI, "Electricity and Life: Volta's Path to the Battery", *Historical Studies in the Physical Sciences*, 21:1 (1990), pp. 123-59, on pp. 143-52; "brain in [his] fingertips" is the apt metaphor of D.J. de Solla Price, quoted by Pancaldi, *ibid.*, p. 152.

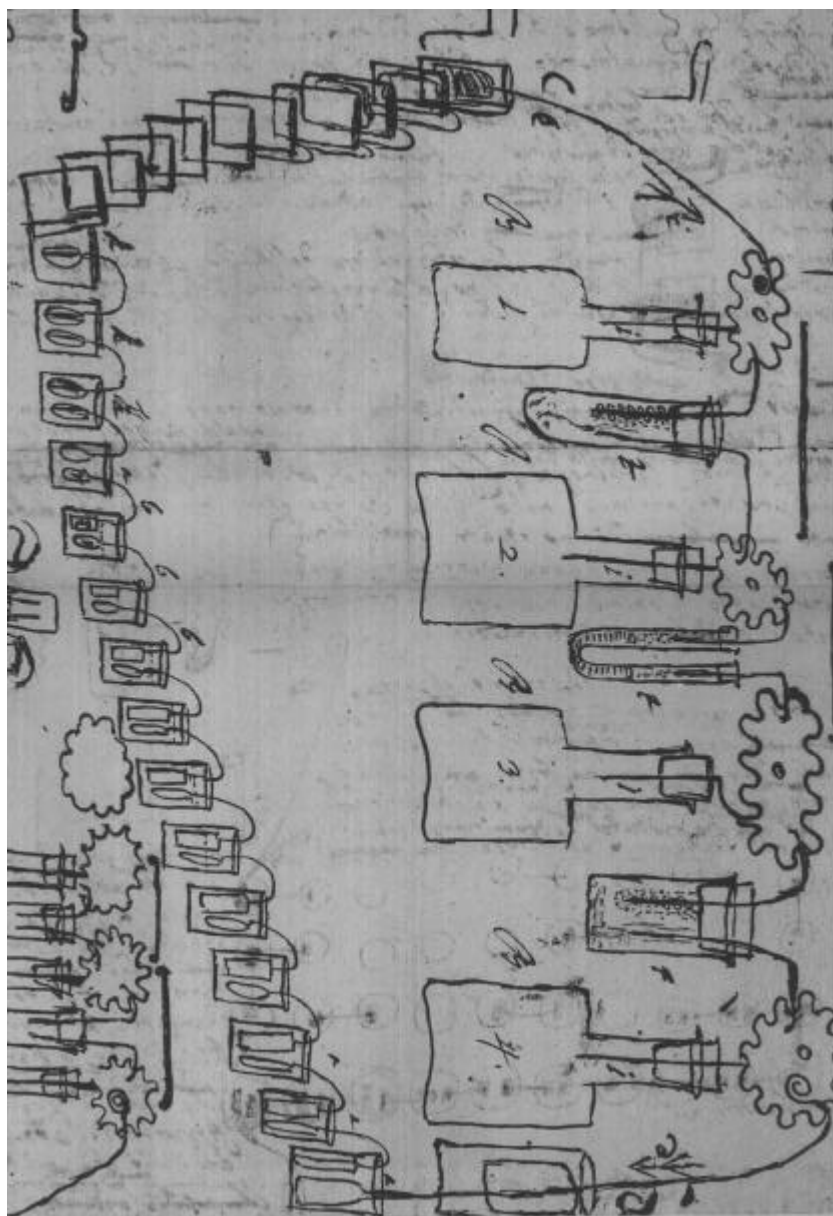


Figure 7 Addition of cells by pencil. *VO*, II, facing p. 149.

moist conductors. Here is Biot's version.⁴³

Let the number of metal plates be $2n$, each of unit capacity, and let the contact emf displace unit quantity of electric fluid from copper to zinc. In the insulated case, let X be the tension of the top zinc disk. Then, counting down, the zincs have tension:

$$X - 1, X - 2 \dots X - (n - 1) \text{ and the coppers } X - 1, X - 2 \dots X - n.$$

The sums of the charges on the zincs and the coppers are:

$$nX - n(n - 1)/2 \text{ and } nX - n(n + 1)/2, \text{ respectively.}$$

Since the total charge on the insulated pile must vanish:

$$2nX - n^2 = 0 \text{ and } X = n/2.$$

The tension of the bottom copper is $X - n = -n/2$. In the grounded case, this tension must be zero; charge flows from the ground sufficient to raise the tension of every element by $n/2$. Hence $X = n$ for a grounded pile. The charge (!) is:

$$n(n + 1)/2 \text{ on the zincs, } n(n - 1)/2 \text{ on the coppers, in all } n^2.$$

A grounded pile holds and delivers a much greater quantity of free electricity than one built up when insulated.⁴⁴ Biot confirmed this insight by drawing charge from the pile into a condensing electroscope.⁴⁵

Biot acknowledged that his account left some obscurities that had to be resolved before the pile could be handled by certain and rigorous calculation. One was the assumption that, irrespective of its position in the pile, the zinc member always received the same quantity of electricity from contact with its copper partner. Only experiment could tell and only then if the experimenter used the Coulomb torsion balance to determine the value of the tension.⁴⁶ In his important textbook of 1816, Biot mentioned that Coulomb himself had – or was said to have – determined that

⁴³ J.B. BIOT, "Rapport sur les expériences du citoyen Volta ... au nom d'une commission composée des citoyens Laplace, Coulomb, Hallé, Monge, Fourcroy, Vauquelin, Pelletan, Charles, Brisson, Sabatier, Guyton et Biot", Institut national des sciences et arts. Sciences mathématiques et physiques, *Mémoires*, 5 (1804), pp. 105-22; also in *VO*, II, pp. 111, 113, 115, 117.

⁴⁴ Cf. Nicholson's paraphrase of Volta's theory: "it is the property of such bodies as differ in their power of conducting electricity, that when they are brought into contact they will occasion a stream of the electric matter. So that if zinc and silver be made to communicate immediately by contact, there will be a place of good conducting energy; and if they be made to communicate mediately by means of water, there will be a place of inferior conducting energy: and whenever this happens there will be a stream or current produced in the general stock of electricity ... this is laid down as a general or simple principle grounded on the phenomena". J.W. NICHOLSON, "Account of the New Electrical or Galvanic Apparatus of Sig. Alex. Volta, and Experiments Performed with the Same", *Journal of Natural Philosophy, Chemistry, and the Arts*, 4 (1801), pp. 179-87, on p. 180.

⁴⁵ BIOT, in *VO*, II, pp. 115-21.

⁴⁶ BIOT, *ibid.*, pp. 111n, 114.

the contact force between elements did not vary with their position within the pile. Biot appears to have accepted this result since his analysis of the pile, “the most beautiful instrument of physics ever conceived”, was the same in 1816 as in 1801.⁴⁷ Though challenged during the 19th century, Biot’s account recurs in Gustav Wiedemann’s definitive *Lehrbuch der Electricität* of 1882.⁴⁸

No doubt, modern theory, which finds the emf of metallic contact in a mismatch of the Fermi levels of the partners, shows that several of Volta’s leading analogies, and the nature of the displacement of charge in the open pile to which they gave rise, were unsound. So what? Modern theorists did not invent the battery.

⁴⁷ J.B. BIOT, *Traité de physique expérimentale et mathématique*, 4 vols., (Paris, 1816), II, pp. 475-6, 477 (quote), 480-1, 490-500.

⁴⁸ G. WIEDEMANN, *Die Lehre von der Electricität*, 4 vols., (Leipzig, 1882-5), I, pp. 183-91, 246-7, 261-4; repeated in WÜLLNER, cit. 39, IV, pp. 486-91.